عدد الوفق = مربع سحرى

لمّا كان اعداد الوفق قد اشتهر من خواصّها ما شغف بسببه كلّ واحد الى معرفة وضعها وكان ذلك بصعب على اكثر الناس طلب منّى بعض اصحابى ان انشىء رسالة موجزة اجمع فيها الضوابط التى يعرف بها وضع وفق جميع الاعداد فانشأتُ هذه الرسالة على مقتضى إلتماسه ورتبتُها على ستّة فصول.

Because magic squares have become famous on account of their properties, to the extent that everyone is eager to know their construction while for most people this is difficult, some of my friends asked me to write a short treatise in which I should sum up the rules whereby a magic square of any order might be constructed. I therefore wrote this treatise in answer to their request. I have divided it into six sections.

واعلم ان الوفق الذى قد اشتهر بالخواصّ انّما هو الوفق التامّ وهو الذى اذا حذف منه ايّ دور كان من الادوار الخارجة الاوّل فالاوّل يكون الوفق فى الباقى حاصلًا اى تكون سطور الباقى طولًا وعرضًا وقطرًا متساويةً بما فيها من العدد ايضًا ونحن نبيّن فى هذه الرسالة طريق وضعه.

You are to know that the magic square which has become famous on account of its properties is in fact the complete magic square. It is that in which, when each outer border is removed in turn, a magic square is found in the remainder; that is, the rows in the remainder contain once again an equal quantity vertically, horizontally, and diagonally. We shall explain in this treatise a method for constructing such a square.

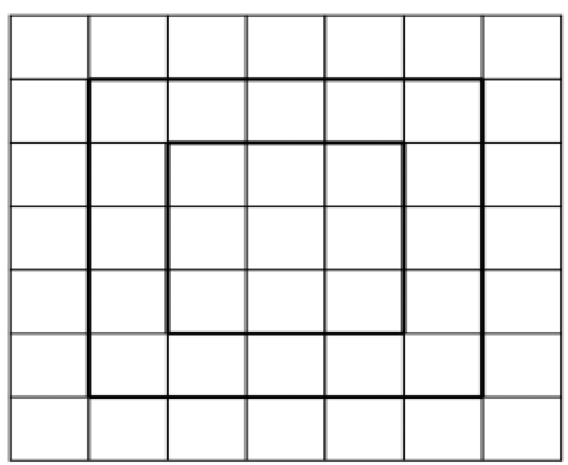
odd (فرد) : $n = 2k + 1 \quad (n = 3,\ 5,\ 7,\ 9,\ 11,\ \dots)$

even (زوج) : $n = 2k$

- evenly odd (زوج فرد) : $n = 4k{+}2 \quad (n = 6,\ 10,\ 14,\ 18,\ \dots)$
- evenly even (زوج زوج) : $n = 4k \quad (n = 4,\ 8,\ 12,\ 16,\ \dots)$

$$M_n = \frac{n\,(n^2+1)}{2}$$

$$n^2 + 1$$



$M_7 = \frac{7\,(7^2+1)}{2} = 175$

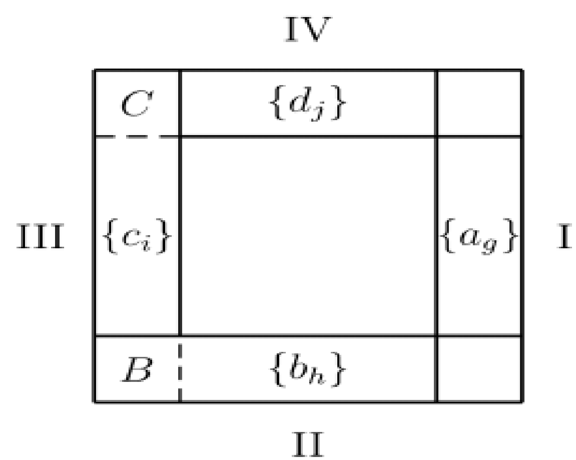
$7^2 + 1 = 50$

١٠	٤٥	٤٤	٧	١١	١٢	٤٦
٩						٤١
٨						٤٢
٤٩						١
٤٨						٢
٤٧						٣
٤	٥	٦	٤٣	٣٩	٣٨	٤٠

٤	٣	٨
٩	٥	١
٢	٧	٦

١٠	٤٥	٤٤	**٧**	**١١**	**١٢**	٤٦
٩	**١٩**	٣٤	**١٧**	**٢٠**	٣٥	٤١
٨	**١٨**	**٢٤**	**٢٣**	٢٨	٣٢	٤٢
٤٩	٣٧	٢٩	**٢٥**	**٢١**	**١٣**	**١**
٤٨	٣٦	**٢٢**	٢٧	٢٦	**١٤**	**٢**
٤٧	**١٥**	**١٦**	٣٣	٣٠	٣١	**٣**
٤	**٥**	**٦**	٤٣	٣٩	٣٨	٤٠

10	45	44	**7**	**11**	**12**	46
9	**19**	34	**17**	**20**	35	41
8	**18**	**24**	**23**	28	32	42
49	37	29	**25**	**21**	**13**	**1**
48	36	**22**	27	26	**14**	**2**
47	**15**	**16**	33	30	31	**3**
4	**5**	**6**	43	39	38	40

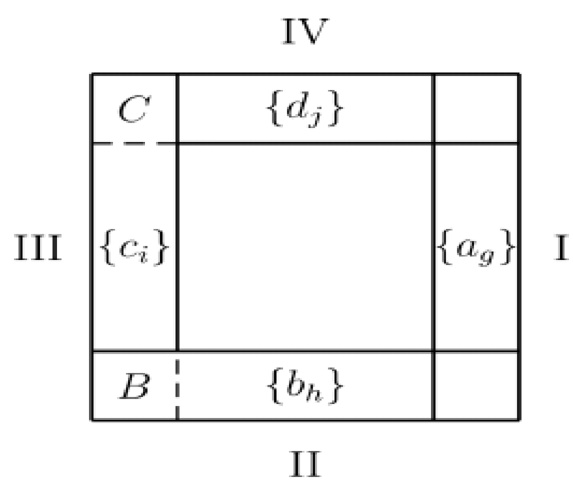


$$n = 2k + 1$$

$$2(2k+1) + 2(2k-1) = 8k$$

$$\sum_{1}^{k} a_g, \quad \sum_{1}^{k} b_h, \quad \sum_{1}^{k} c_i, \quad \sum_{1}^{k} d_j$$

$\{a_g\}$	$1,\ 2,\ \ldots,\ k$	with sum	$\frac{k(k+1)}{2}$
$\{b_h\}$	$k+1,\ k+2,\ \ldots,\ 2k$	with sum	$k^2 + \frac{k(k+1)}{2}$
$\{c_i\}$	$2k+1,\ 2k+2,\ \ldots,\ 3k$	with sum	$2k^2 + \frac{k(k+1)}{2}$
$\{d_j\}$	$3k+1,\ 3k+2,\ \ldots,\ 4k$	with sum	$3k^2 + \frac{k(k+1)}{2}$.



$$B = b_1 + x, \qquad C = c_1 + y$$

$$M_n = \frac{n}{2}(n^2+1) = \left(\frac{2k+1}{2}\right)(n^2+1) = \left(k + \frac{1}{2}\right)(n^2+1)$$

$$\sum_{1}^{g} a_g + (n^2+1) - (b_1+x) + (n^2+1) - (c_1+y) + (k-1)(n^2+1) - \sum_{i\neq 1} c_i$$

$$= \frac{k(k+1)}{2} + (k+1)(n^2+1) - (k+1) - x - y - 2k^2 - \frac{k(k+1)}{2}$$

$$= (k+1)(n^2+1) - x - y - 2k^2 - k - 1 \quad = \quad M_n = \left(k+\frac{1}{2}\right)(n^2+1)$$

$$\frac{1}{2}(n^2+1) = x + y + 2k^2 + k + 1$$

$$2k^2 + 2k + 1 = x + y + 2k^2 + k + 1$$

$$x + y = k$$

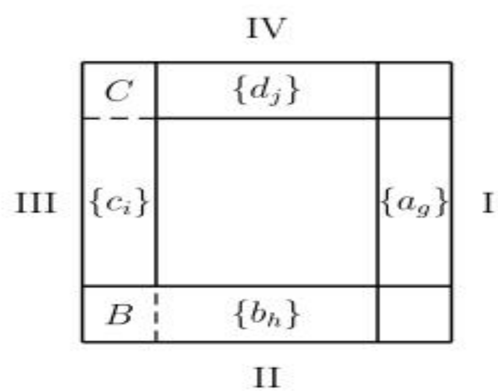
Choose : $x = 0$, thus b_1 in one angle
$y = k$, thus $3k+1 = d_1$ in one angle as one c_i
therefore, c_1 takes the place of one d_j

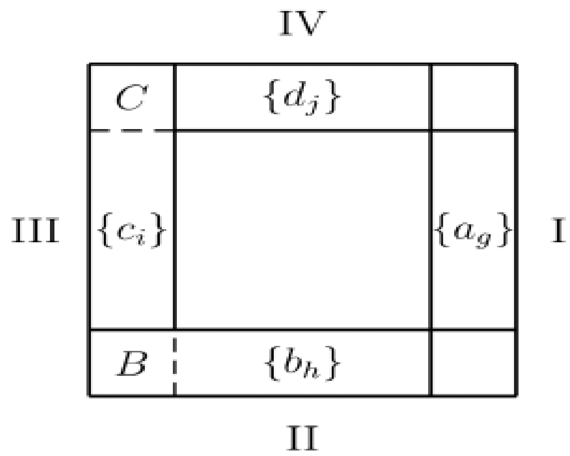
$\{a_g\}$ $1, 2, ..., k$

$\{b_h\}$ $\mathbf{k+1}, k+2, ..., 2k$

$\{c_i'\}$ $2k+2, ..., 3k, \underline{\mathbf{3k+1}}$

$\{d_j'\}$ $\underline{2k+1}, 3k+2, ..., 4k.$





$\{a_g\}$	$1,\ 2,\ \ldots,\ k$
$\{b_h\}$	$\mathbf{k+1},\ k+2,\ \ldots,\ 2k$
$\{c'_i\}$	$2k+2,\ \ldots,\ 3k,\ \underline{\mathbf{3k+1}}$
$\{d'_j\}$	$\underline{2k+1},\ 3k+2,\ \ldots,\ 4k$

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4	**5**	**6**	43	39	38	40

$$n = 4k \qquad (k \geq 2)$$

٣							٢
٥							
							٦
					٤	١	

٣	١٠	٥٦	٥٧	٧	٦١	٦٤	٢
٥							٦٠
٥٩							٦
٥٤							١١
١٢							٥٣
١٣							٥٢
٥١							١٤
٦٣	٥٥	٩	٨	٥٨	٤	١	٦٢

• Fill in each row a *small* number of cells, including the angular cells

• Do it with the *first consecutive numbers*

• Obtain the *sum due*

• The number of cells remaining empty in each row must be *divisible by 4*

$$M_n = \frac{n\,(n^2+1)}{2}$$

$$n^2+1$$

	a	$n^2{+}1{-}(a{+}s)$	$n^2{+}1{-}(a{+}2s)$	$a{+}3s$		$\Longrightarrow$	$2(n^2{+}1)$
	$n^2{+}1{-}a$	$a{+}s$	$a{+}2s$	$n^2{+}1{-}(a{+}3s)$		$\Longrightarrow$	$2(n^2{+}1)$

$$n = 4k + 2$$

٤	١٠				١
٢					
					٦
					٨
٩					
		٧	٥	٣	

٦		٨			١
٣					
					٥
٧					
					١٠
	٩		٤	٢	

٤	١٠	٣٠	٣٢	٣٤	١
٣١					٦
٢					٣٥
٢٩					٨
٩					٢٨
٣٦	٢٧	٧	٥	٣	٣

٦	٨	٢٨	٣٣	٣٥	١
٣					٣٤
٧					٣٠
٣٢					٥
٢٧					١٠
٣٦	٢٩	٩	٤	٢	٣١

٤	١٤	٨٨	٨٩	١١	١٠	٩٤	٩٦	٩٨	١
٢									٩٩
٩٥									٦
٩٣									٨
٩									٩٢
٨٦									١٥
١٦									٨٥
١٧									٨٤
٨٣									١٨
١٠٠	٨٧	١٣	١٢	٩٠	٩١	٧	٥	٣	٩٧

c								b	a
									g
									h
i									
j									
						f	e	d	

$$\begin{cases} a+b+c & = & d+e+f \\ a+g+h & = & c+i+j \end{cases}$$

4	**10**	*30*	*32*	*34*	**1**
2					*35*
31					**6**
29					**8**
9					*28*
36	*27*	**7**	**5**	**3**	*33*

6	*28*	**8**	*33*	*35*	**1**
3					*34*
32					**5**
7					*30*
27					**10**
36	**9**	*29*	**4**	**2**	*31*

٨			١
	٢	٧	
٣			٦
	٥	٤	

٨	١١	١٤	١
١٣	٢	٧	١٢
٣	١٦	٩	٦
١٠	٥	٤	١٥

٣	١٤	١٣	١٣٣	١٣٤	١٣٥	١٣٦	٨	٧	١٤١	١٤٤	٢
٥	٢٦	٣٦	١١٠	١١١	٣٣	٣٢	١١٦	١١٨	١٢٠	٢٣	١٤٠
١٣٩	٢٤	٤٣	٥٠	٩٦	٩٧	٤٧	١٠١	١٠٤	٤٢	١٢١	٦
١٣٠	١١٧	٤٥	٥٨	٦٤	٨٤	٨٦	٨٨	٥٥	١٠٠	٢٨	١٥
١٢٩	١١٥	٩٩	٥٦	٧٢	٧٥	٧٨	٦٥	٨٩	٤٦	٣٠	١٦
١٧	٣١	٩٤	٨٥	٧٧	٦٦	٧١	٧٦	٦٠	٥١	١١٤	١٢٨
١٨	١٠٨	٥٢	٨٣	٦٧	٨٠	٧٣	٧٠	٦٢	٩٣	٣٧	١٢٧
١٩	٣٨	٥٣	٦٣	٧٤	٦٩	٦٨	٧٩	٨٢	٩٢	١٠٧	١٢٦
٢٠	٣٩	٩١	٩٠	٨١	٦١	٥٩	٥٧	٨٧	٥٤	١٠٦	١٢٥
١٢٤	١٠٥	١٠٣	٩٥	٤٩	٤٨	٩٨	٤٤	٤١	١٠٢	٤٠	٢١
١٢٣	١٢٢	١٠٩	٣٥	٣٤	١١٢	١١٣	٢٩	٢٧	٢٥	١١٩	٢٢
١٤٣	١٣١	١٣٢	١٢	١١	١٠	٩	١٣٧	١٣٨	٤	١	١٤٢

d					c	b	a
							i
j							
	h	g	f	e			

$$\begin{cases} a+b+c+d = e+f+g+h \\ a+i = d+j, \end{cases}$$